HYDRO DYNAMICS

FUNDAMENTALS OF FLUID FLOW

Discharge or Flow Rate, Q

The amount of fluid passing a section of a stream in unit time is called the *discharge*. If *v* is the mean velocity and *A* is the cross sectional area, the discharge *Q* is defined by $Q = Av$ which is known as *volume flow rate*. Discharge is also expressed as *mass flow rate* and *weight flow rate*.

Volume flow rate, $Q = Av$

Mass flow rate, $M = \rho Q$

Weight flow rate, $W = \gamma Q$

Where:

Q = discharge in m^3/sec or ft^3/sec

A = cross-sectional area of flow in m^2 or ft^2

v = mean velocity of flow in m/sec or ft/sec

ρ = mass density of fluid in kg/m^3 or $slugs/ft^3$

γ = unit weight of fluid in N/m^3 or lb/ft^3

Fundamentals of Fluid Flow

Discharge or Flow Rate, Q

Definition of Terms

Fluid flow may be *steady* or *unsteady*; *uniform* or *non-uniform*; *continuous*; *laminar* or *turbulent*; *one-dimensional*, *two-dimensional* or *three-dimensional*; and *rotational* or *irrotational*.

Steady Flow

Steady flow occurs if the discharge Q passing a given cross section of a stream is constant with time, otherwise the flow is unsteady.

Uniform Flow

The flow is said to be uniform if, with steady flow for a given length, or reach, of a stream, the average velocity at every cross-section is the same. Uniform flow usually occurs to incompressible fluids flowing in a stream of constant cross section. In streams where velocity and cross section changes, the flow is said to be non-uniform.

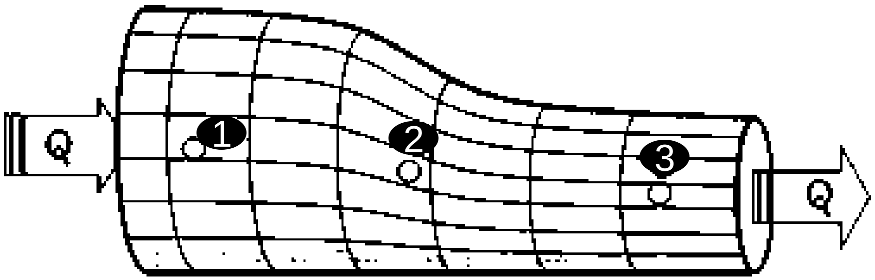
Fundamentals of Fluid Flow

Discharge or Flow Rate, Q

Definition of Terms

Continuous Flow

By the principle of conservation of mass, continuous flow occurs when at any time, the discharge Q at every section of the stream is the same.



Continuity Equations

For incompressible fluids:

$$Q = A_1 v_1 = A_2 v_2 = A_3 v_3 = \text{ constant}$$

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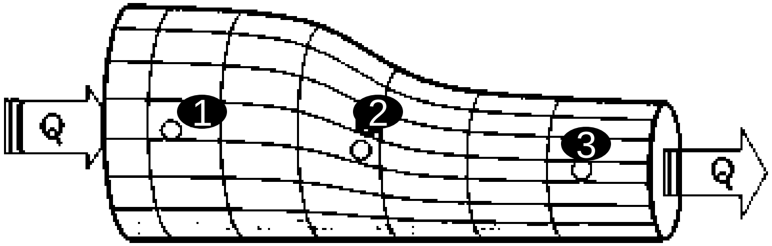


For compressible fluids:

$$M = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \rho_3 A_3 v_3 = \text{ constant}$$

or

$$W = \gamma_1 A_1 v_1 = \gamma_2 A_2 v_2 = \gamma_3 A_3 v_3 = \text{ constant}$$

Discharge or Flow Rate, Q

Definition of Terms

Laminar Flow

Flow is said to be laminar when the paths of the individual particles do not cross or intersect. By many careful experiments to commercial pipes of circular cross section, the flow is laminar when the Reynolds' number Re is less than 2100.

Turbulent Flow

The flow is said to be turbulent when its path lines are irregular curves and continuously cross each other. The paths of particles of a stream flowing with turbulent motion are neither parallel nor fixed but it aggregates to forward motion of the entire stream. Reynolds' number greater than 2100 normally defines turbulent flow but in highly controlled environment such as laboratories, laminar flow can be maintained up to values of Re as high as 50,000. However, it is very unlikely that such condition can occur in the practice.

Discharge or Flow Rate, Q

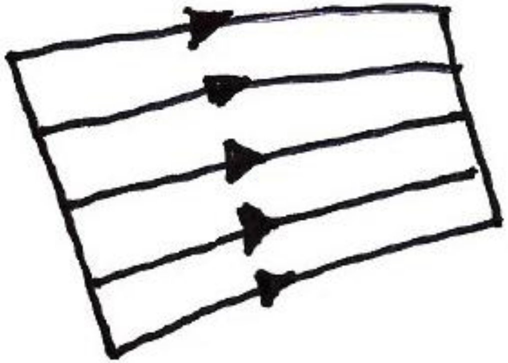
Definition of Terms

One-Dimensional Flow

This occurs when in an incompressible fluid, the direction and magnitude of the velocity at all points are identical.

Two-Dimensional Flow

This occurs when the fluid particles move in planes or parallel planes and the streamline patterns are identical in each plane.



Fundamentals of Fluid Flow

Discharge or Flow Rate, Q

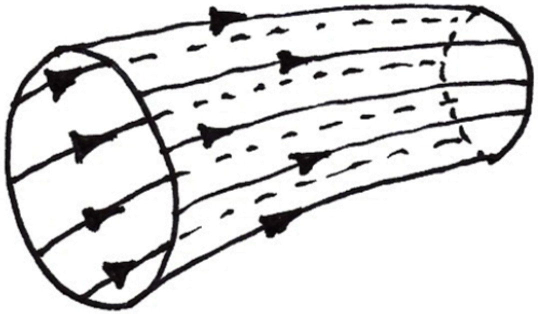
Definition of Terms

Streamlines

These are imaginary curves drawn through a fluid to indicate the direction of motion in various sections of the flow of the fluid system.

Streamtubes

These represents elementary portions of a flowing fluid bounded by a group of streamlines which confine the flow.



Problem Set 12

Problem 1

Compute the discharge of water through 75 *mm* pipe if the mean velocity is 2.5 *m/sec*.

Solution:

$$Q = vA = 2.5[\,\tfrac{1}{4}\pi(0.075^2)\,]$$

$$Q = 0.011\ \text{m}^3/\text{sec} \qquad \textit{answer}$$

Problem Set 12

Problem 2

The discharge of air through a 600-*mm* pipe is 4 m^3/sec. Compute the mean velocity in *m/sec*.

Solution:

$$Q = vA$$

$$4 = v[\,\tfrac{1}{4}\pi(0.6^2)\,]$$

$$v = 14.15\ \text{m/sec} \qquad \textit{answer}$$

Problem Set 12

Problem 3

A pipe line consists of successive lengths of 380-*mm*, 300-*mm*, and 250-*mm* pipe. With a continuous flow through the line of 250 *lit/sec* of water, compute the mean velocity in each size of pipe.

Solution:

Problem Set 12

Problem 3

For continuous flow

$$Q_1 = Q_2 = Q_3 = Q$$

$$v_1 A_1 = v_2 A_2 = v_3 A_3 = Q$$

$$v_1[\,\tfrac{1}{4}\pi(0.380^2)\,] = v_2[\,\tfrac{1}{4}\pi(0.300^2)\,] = v_3[\,\tfrac{1}{4}\pi(0.250^2)\,] = 0.25$$

$$v_1 = \frac{4(0.25)}{\pi(0.380^2)} = 2.20 \ \text{m/sec} \qquad \textit{answer}$$

$$v_2 = \frac{4(0.25)}{\pi(0.300^2)} = 3.54 \ \text{m/sec} \qquad \textit{answer}$$

$$v_3 = \frac{4(0.25)}{\pi(0.250^2)} = 5.09 \ \text{m/sec} \qquad \textit{answer}$$

Problem Set 12

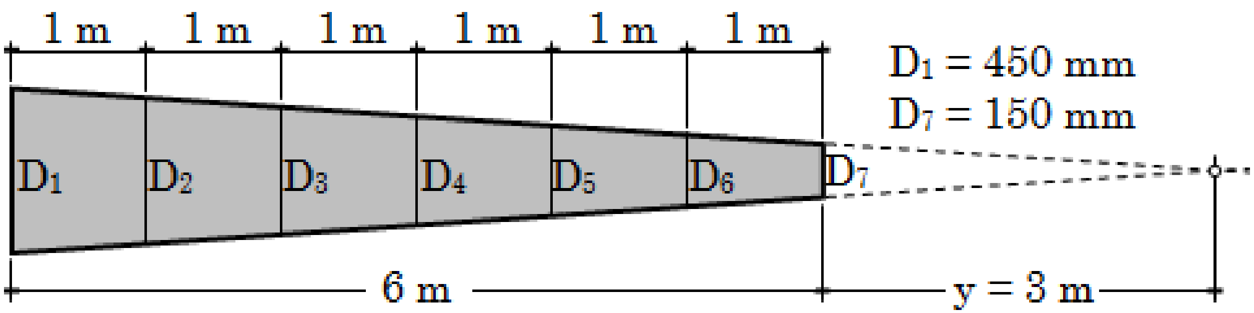
Problem 4

The diameter of a 6-m length pipe decreases uniformly from 450 mm to 150 mm. With a flow of 0.15 m^3/sec of oil, compute the mean velocity at cross section 1 m apart. Plot the velocity as ordinate against length as abscissa.

Solution:

Problem Set 12

Problem 4



Solving for y by ratio and proportion

$$\frac{y}{D_7} = \frac{y+6}{D_1}$$

$$D_1 y = D_7(y+6)$$

$$450y = 150(y+6)$$

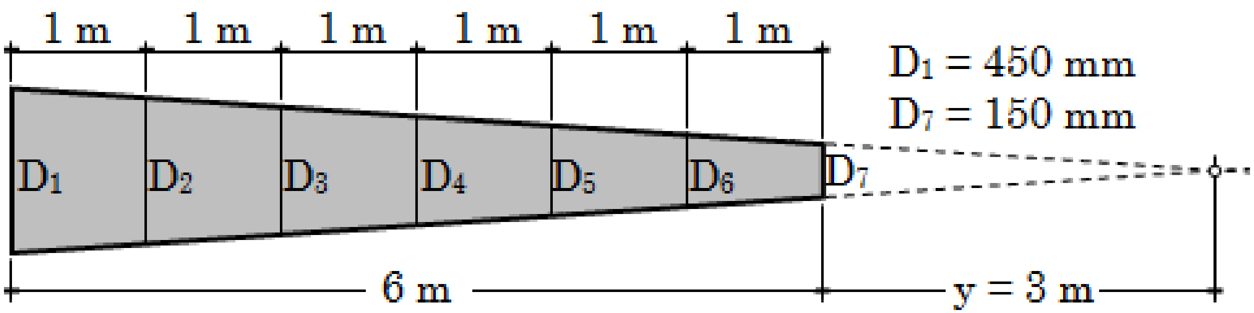
$$300y = 900$$

$$y = 3 \text{ m}$$

Fundamentals of Fluid Flow

Problem Set 12

Problem 4



Solving for diameters by ratio and proportion

$$\frac{D_2}{5+y} = \frac{D_3}{4+y} = \frac{D_4}{3+y} = \frac{D_5}{2+y} = \frac{D_6}{1+y} = \frac{D_7}{y}$$

$$\frac{D_2}{5+3} = \frac{D_3}{4+3} = \frac{D_4}{3+3} = \frac{D_5}{2+3} = \frac{D_6}{1+3} = \frac{150}{3}$$

$$\frac{D_2}{8} = \frac{D_3}{7} = \frac{D_4}{6} = \frac{D_5}{5} = \frac{D_6}{4} = 50$$

$$D_2 = 50(8) = 400 \text{ mm}$$

$$D_3 = 50(7) = 350 \text{ mm}$$

$$D_4 = 50(6) = 300 \text{ mm}$$

$$D_5 = 50(5) = 250 \text{ mm}$$

$$D_6 = 50(4) = 200 \text{ mm}$$

Fundamentals of Fluid Flow

Problem Set 12

Problem 4

Formula for velocity of flow

$$Q = vA$$

$$v = Q/A$$

Velocity at sections 1-m apart

$$v_1 = 0.15/[\,\tfrac{1}{4}\pi(0.450^2)\,] = 0.943\ \text{m/sec}$$

$$v_2 = 0.15/[\,\tfrac{1}{4}\pi(0.400^2)\,] = 1.194\ \text{m/sec}$$

$$v_3 = 0.15/[\,\tfrac{1}{4}\pi(0.350^2)\,] = 1.559\ \text{m/sec}$$

$$v_4 = 0.15/[\,\tfrac{1}{4}\pi(0.300^2)\,] = 2.122\ \text{m/sec}$$

$$v_5 = 0.15/[\,\tfrac{1}{4}\pi(0.250^2)\,] = 3.056\ \text{m/sec}$$

$$v_6 = 0.15/[\,\tfrac{1}{4}\pi(0.200^2)\,] = 4.775\ \text{m/sec}$$

$$v_7 = 0.15/[\,\tfrac{1}{4}\pi(0.150^2)\,] = 8.488\ \text{m/sec}$$

Fundamentals of Fluid Flow

Problem Set 12

Problem 4

Graph of velocity of flow versus length of pipe (plotted in MS Excel)